# Faster accurate reflections throught quadric mirrors 

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Figure 1: The light ray reflection by a quadratic surface (sphere in this example).

## 1 Introduction

Reflectors attract the attention of people since they reflect discontinuous images of the world and often provide unexpected information of a non-direct field of view. This is why reflections still have a lot of research attention in rendering of images in computer graphics, computer vision and optics, amongst other fields.

In a generic combination of camera (image plane) and reflectors there isn't any expression that gives us the point on the reflector surface where the light ray is reflected on the image direction (this expression, when it exists, is called projection model). The non existence of this projection model limits the rendering process to use back tracing with the well-known advantages and drawbacks. The use of forward methods to project points (very popular for the use with global illumination and other rendering techniques) is achieved by either using Snell Law or Fermat Principle (which give accurate results) or by introducing approximations on the reflections [Szirmay-Kalos et al. 2009; Estalella et al. 2006; Roger and Holzschuch 2006]. The necessary trade-off between performance and accuracy can severely affect the quality of the resulting images.

Goncalves [Goncalves 2010] recently proved that when the reflector is a second order quadratic surface (includes spheres, ellipsoids, paraboloids and hyperboloids) the projection model that relates a 3D world point with its corresponding image point (knowing the reflection point) can be searched for in a unidimensional curve on the reflector surface. This result was proved to be a key idea to improve the performance of forward projection of world point to the image. Particularly it has been proved that the proposed method can compute the reflection point with a one-order of magnitude quicker method than those that usually use Fermat Principle or Snell Law.

## 2 Our approach

Suppose a 3D world point located at the source $\mathbf{S}$ and a viewing eye at the target $\mathbf{T}$ as showed in right part of figure 1. If the reflector surface is a quadratic equation expressed by the matrix $\mathbf{Q}$, then it has been proved by Goncalves [Goncalves 2010] that the reflection point belongs to a curve on the reflector surface given by the intersection of quadric $\mathbf{Q}$ and quadric $\mathbf{A}=f(\mathbf{Q}, \mathbf{S}, \mathbf{T})$. The curve is

[^0]then searched for the point where incident and reflected angles are equal (Snell Law) or for the point that makes the light travel the fastest path (Fermat Principle)

In practice it means that the reflection point on the surface (essential for rendering with reflections), is searched for in a unidimensional curve instead of being searched for in a multidimensional space.

Our experimental tests show that when compared with the classical approaches using Snell Law or Fermat Principle, our method is one order of magnitude quicker, while maintaining the accuracy. It is also well-known that non linear optimization algorithms in one dimension converge much quicker than in a multi dimensional space.

Figure 1 shows an example of a rendering using our reflection algorithm with highly accurate and realistic reflection information.

For future directions we want to compare our algorithm with the mesh-based methods that usually obtain better performances with low accuracy and sometimes introducing undesirable artifacts. For that, we want to first adapt our approach to use a graphic processing unit (GPU) and to extend the reflectors to other surfaces that can be modeled or approximated by quadratic equations.

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## References

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